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I Will If You Will

An Examination of Patterns in Group Behavior

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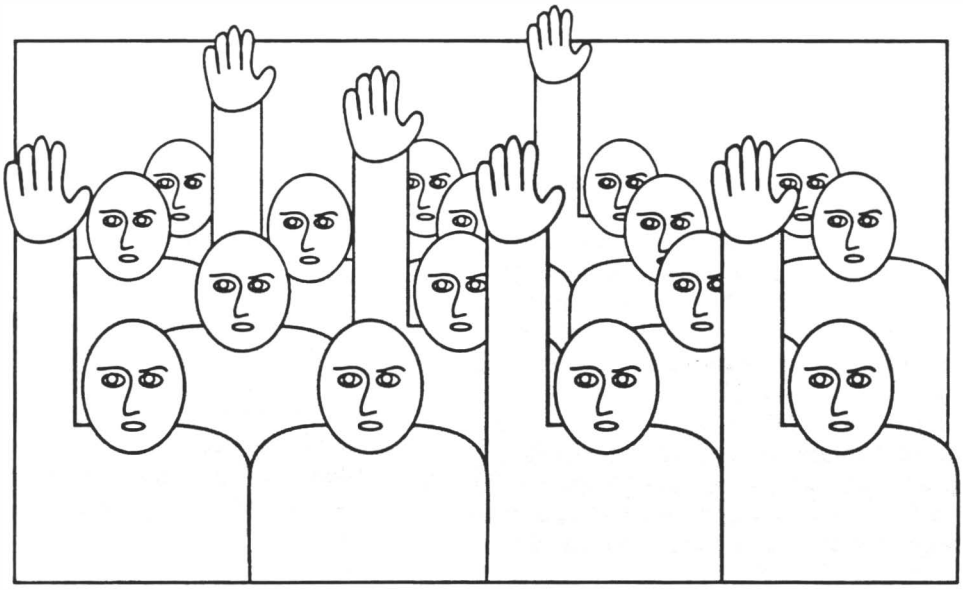

Has a situation like the following one ever happened to you?

Situation A:
Tom and a couple of friends are discussing whether to try out for the junior class play. Tom says, "I will if one of you guys will." Dick says, "I will if you two guys will." But Harry says, "No way! I'm not going to do it." The result of the discussion is that none of the tree tries out for the play.

Now consider a slightly different situation.

Situation B:
Tom, Dick, Harry, and a fourth friend, Fred, are discussing whether to try out for the junior class play. As before, Tom says, "I will if one of you guys will." Dick says, "I will if two of you guys will." Harry says, "No way! I'm not going to do it." Fred says, "I'm going to try out no matter what the rest of you do." The result of this discussion is that Fred, Tom, and Dick all try out.

STOP AND THINK

1. Review Situation B. What is the chain of reasoning that convinces Tom to try out for the junior class play? What is the chain of reasoning that convinces Dick to try out? Why do these two not try out in Situation A?

2. What are situations in your experience in which your choice of whether or not to do something has been influenced by how many of your friends will join you in doing it? What was the outcome of each situation: how many of you ended up doing it? (In the paragraphs to follow we will be analyzing such situations. You will find it interesting to try to apply these methods to analyze your own experiences.)

Situations A and B are simple and no mathematical terminology is needed to understand them and to see the role that Fred played in changing the outcome for Tom and Dick. Nevertheless, these examples provide a setting in which to introduce some mathematical ideas that can then be applied to complex situations.

Situations A and B and other examples we will examine have some things in common. All involve a group of people in which each member is considering whether or not to participate in a certain activity. We will refer to the group being studied as the *target group*, and the activity for which participation is considered as the *target behavior*. Each potential participant either declares, "I will not participate" or supplies a response, called a *threshold*, to fill in the blank in the following statement.

I will participate if I will be part of a group of size at least _____.

That is, a person's *threshold* is the size of the smallest group of participants of which he or she would be willing to be a part. Anyone who will not participate at all is said to have no threshold. The examination of the combined effects of individual thresholds and the group behavior that results is called *threshold analysis*.

For Situation B with the four friends discussing whether to try out for the junior class play,

Fred has a threshold equal to 1;
Tom has a threshold equal to 2;
Dick has a threshold equal to 3; and
Harry has no threshold.

In both of Situations A and B, Tom, Dick, and Harry provided identical information about thresholds. The difference, which affected the outcome for Tom and Dick, was the presence of Fred with a threshold of 1. Tables 1(a) and 1(b) summarize the threshold information for these situations.

Table 1(a)

Threshold	Number holding threshold
1	0
2	1
3	1
None	1

Threshold information for Situation A (target group: Tom, Dick, Harry; target behavior: trying out for junior class play).

Table 1(b)

Threshold	Number holding threshold
1	1
2	1
3	1
None	1

Threshold information for Situation B (target group: Fred, Tom, Dick, Harry; target behavior: trying out for junior class play).

The different outcomes of Situations A and B result from the different cumulative effects when threshold values are combined. When Tom learned that Fred would try out for the play, he saw that his threshold of 2 was met, and he also decided to try out. When Dick learned of Fred's and Tom's intentions, he saw that his threshold of 3 was met, and he decided to try out.

Another way to interpret the situation is to observe that if an individual has a threshold of t, then that individual will be satisfied to participate as part of a group of size t or greater. Thus, the number of people satisfied with a given threshold t is the total of all holding thresholds up to and including t. These totals for Situations A and B are shown in Tables 2(a) and 2(b). These tables are like Tables 1(a) and 1(b) except for the addition of a third column. Each entry in the third column has been obtained by summing all entries in the second column up to and including the one opposite it.

Table 2(a)

Threshold	Number holding threshold	Total number satisfied by threshold
1	0	0
2	1	1
3	1	2
None	1	

Cumulative threshold information for Situation A (target group: Tom, Dick, Harry; target behavior: trying out for junior class play).

Table 2(b)

Threshold	Number holding threshold	Total number satisfied by threshold
1	1	1
2	1	2
3	1	3
None	1	

Cumulative threshold information for Situation B (target group: Fred, Tom, Dick, Harry; target behavior: trying out for junior class play).

The final step in our examination of Situations A and B will involve graphing. For each of the situations we draw a horizontal scale that displays the possible threshold values. Above each threshold we construct a line segment whose height is equal to the total number satisfied by the given threshold. Figures 1a and 1b display the cumulative threshold information from Tables 2(a) and 2(b).

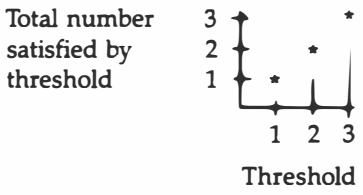

Figure 1a. Threshold graph for Situation A (target group: Tom, Dick, Harry; target behavior: trying out for junior class play).

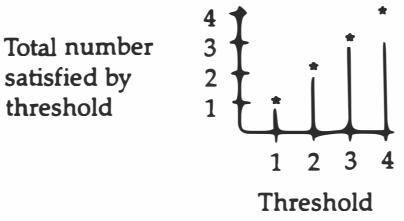

Figure 1b. Threshold graph for Situation B (target group: Fred, Tom, Dick, Harry; target behavior: trying out for junior class play).

In the graphs of Figure 1a and Figure 1b, the points (1,1), (2,2), (3,3), and (4,4) have been indicated using *. These points with equal coordinates are called *equilibrium points;* each is a point at which the threshold is equal to the number of persons willing to participate in the target behavior. When the vertical segment for a given threshold reaches (or passes) an equilibrium point then the group of people represented by the segment is willing to participate in the target behavior. When the vertical segment does not reach as high as the equilibrium point, the height of the equilibrium point represents a participation level not achievable by the group represented by that vertical segment. Thus the graphs display what we have already observed about Situations A and B — namely, that in Situation A the result will be no participation but in Situation B, groups of size 1, 2, or 3 are willing to participate.

You Try It #1: Southern High School has recently started a girls' track and field program. Five girls who are friends have been discussing whether to go out for track. Two of them (Amy and Beth) each say that they'll go out for track if at least two other friends will; each of the others (Carol, Denise, and Ellen) says she is willing to go out for the sport only if all four others do.

(a) Complete the following table of threshold values.

Threshold	Number holding threshold	Total number satisfied by threshold
1		
2		
3		
4		
5		
None		

(b) Draw a graph of the threshold information.

(c) Analyze this situation. How many girls do you expect to go out for track? Explain how you arrived at your answer.

Individual behavior is influenced by what others do in large groups as well as in small ones. For example, some drivers on an interstate highway will exceed the speed limit only if many other drivers are doing so. Those against nuclear weapons may join a protest march only if they will be part of a sizable group. Some students may adopt a particular hair style only if enough others are wearing it so that they will not feel conspicuous. Our next examples will illustrate the application of threshold analysis to larger groups.

Consider the following situation. The Conestoga High School football team has just begun its fall practices and among the 40 team members are Lee and Dan, who have just returned from a vacation at the shore where almost all the guys were wearing very short haircuts. They began trying to convince their teammates to join them in getting short trims. Suppose the thresholds of team members are those displayed in Table 3(a). (Observe that we have reported all thresholds in terms of "round" numbers. It is not necessary to do this, but often it is convenient to do so.) What is likely to happen to hairstyles on the Conestoga team?

Table 3(a)

Threshold	Number holding threshold
10	5
20	30
30	5
40	0
None	0

Threshold information for Conestoga football team (target behavior: get short haircut).

To analyze this situation we first create Table 3(b) by adding to Table 3(a) a third column that contains the total number satisfied by each threshold. We then draw a threshold graph; see Figure 2.

Table 3(b)

Threshold	Number holding threshold	Total number satisfied by threshold
10	5	5
20	30	35
30	5	40
40	0	40
None		

Cumulative threshold information for Conestoga football team (target behavior: get short haircut).

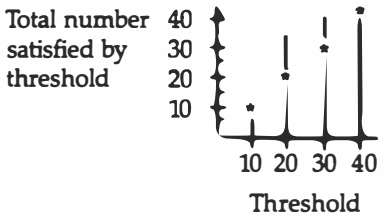

Figure 2. Threshold graph for short haircuts for Conestoga football team.

From the graph we see that the three equilibrium points of (20,20), (30,30), and (40,40) are reached or passed by the vertical line segments. Thus the groups of size 35 or 40 that are represented by those segments are willing to participate in the target behavior of wearing short haircuts.

However, because this group is so large that we do not expect all of its members to know each other's views, the threshold information alone does not determine what will happen. Left to themselves, the team members may never know the attitudes of enough others to cause them to get short cuts. On the other hand, if Lee and Dan are anxious to have the style catch on and do a lot of talking with team members, they may be able to publicize group attitudes and to persuade people that their thresholds will be met and to induce group participation.

You Try It #2: Analyze the following variation of the Conestoga football team haircut threshold information.

Threshold	Number holding threshold	Total number satisfied by threshold
10	10	
20	0	
30	20	
40	0	
None	0	

A periodic difficulty that many communities face is drought and a resulting shortage in water supply. At such times, these communities are likely to impose restrictions on water use: lawns may not be watered, cars may not be washed, and so on. Such restrictions usually are hard to enforce. Furthermore, if one person is seen watering a lawn, neighbors who have been carefully conserving may wonder, "What good does it do for me to conserve water when others don't?" and then may violate the restrictions themselves.

Consider the community of Millville, where over 1500 families draw their water from a small reservoir. Almost all Millville residents (98%) support the water-use restrictions that have been imposed by the borough council. Threshold values are given in Table 4. (Because of the large number of members in the target group, it is convenient to give threshold values as percentages rather than as numbers.)

Table 4

Threshold	Percent holding threshold	Total percent satisfied by threshold
10%	0	0
20%	0	0
30%	0	0
40%	0	0
50%	40	40
60%	0	40
70%	20	60
80%	0	60
90%	20	80
100%	18	98
None	2	

Threshold information for Millville residents (target behavior: water conservation).

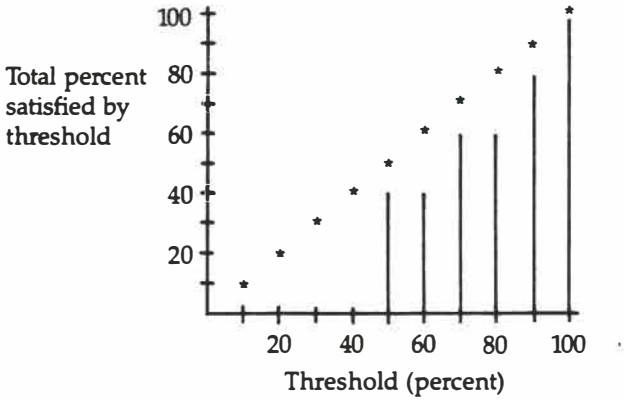

Figure 3. Threshold graph for Millville residents (target behavior: water conservation).

The graph of Figure 3 reveals the undesirable information that there is not a group of Millville residents large enough to meet the thresholds that have been expressed and to conserve. While 98% of the residents show some support of conservation, their thresholds are so high that a cooperative effort at conservation seems to elude them.

If the Millville community leaders believe that conservation is very important, this threshold analysis can be helpful to them. The difficulty is not that Millville residents fail to support conservation but that many people's thresholds are just a bit too high to result in the cooperative effort needed to conserve. A town meeting to explain the importance of conservation might be all that would be needed to shift residents' attitudes a bit and to obtain the needed conservation. (See You Try It #3.)

The threshold information about Millville residents' attitudes toward conservation was invented. For a very large group, it may be very difficult to conduct a survey or to otherwise obtain the information needed for threshold analysis. However, people who understand threshold analysis can use its ideas as a basis for understanding and changing group behavior even when accurate data is unavailable.

It would likely be the case in Millville and in many other large-group situations that group members would know very little about the actual behavior of most others on which their own behavior is based. For example, a Millville resident who sees three neighbors watering their lawns may suppose that many residents are not conserving water and thus may violate conservation restrictions without finding out what the majority of residents are doing. The analyst who studies threshold data must not only consider what the group members' thresholds *are*, but also must consider what others *perceive them to be.*

You Try It #3: Suppose that a town meeting was held and every resident of Millville (except for the 2% who oppose conservation completely) was persuaded to lower his or her conservation threshold by 10%. The results of this change are shown in the table below. Analyze the situation and describe the conservation behavior you think will result.

Threshold	Percent holding threshold	Total percent satisfied by threshold
10%	0	
20%	0	
30%	0	
40%	40	
50%	0	
60%	20	
70%	0	
80%	20	
90%	18	
100%	0	
None	2	

You Try It #4: Merton High School Student Council is considering establishment of a high school honor code. As part of a preliminary investigation, the council took a survey of student attitudes concerning copying of homework. The table below summarizes the information gathered from the 100 Merton seniors. Threshold values were obtained as responses to the statement:

I don't feel bad about copying homework and will do so if I am part of a group of size at least _____.

Analyze the threshold information and discuss the results of your analysis.

Threshold	Number holding threshold	Total number satisfied by threshold
10%	5	
20%	5	
30%	10	
40%	10	
50%	20	
60%	40	
70%	0	
80%	0	
90%	0	
100%	0	
None	10	

After you have completed the You Try It exercises above, you may wish to learn more about threshold analysis. The following references all supply additional information. In Thomas Schelling's book, he discusses threshold analysis using the concept of *critical mass*. The term critical mass has been adopted from nuclear engineering, where it refers to the amount of a radioactive substance necessary to sustain a chain reaction. In Situation B, discussed at the beginning of this lesson, Fred — who would try out for the play even if no one else would — provided a critical mass that led to the additional participation of Tom and Dick. On the other hand, in Situation A, no person provided a critical mass and, as a result, no one decided to try out for the play.

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Project

Threshold analysis is of most interest when we can use it to learn about the behavior of groups to which we belong. Below are suggested steps for you to follow to conduct your own threshold analysis.

Step 1: Choose a group to which you belong and a situation in which members face a choice of engaging (or not engaging) in a particular target behavior. Possible behaviors include:

- attending (or not) a particular event;
- conserving (or not) a valuable resource;
- supporting (or not) a political candidate or issue;
- wearing (or not) a particular type of clothing — such as a tuxedo to a prom or a helmet when cycling;
- conforming (or not) to certain rules — such as liquor and drug laws, parking restrictions, and speed limits;
- joining (or resisting) a particular fad.

Step 2: Survey the members of the target group to learn members' threshold values for the target behavior.

Step 3: Tabulate the survey results and construct a graph.

Step 4: Use the graph to predict the group behavior that is likely to occur.

Step 5: Evaluate the project results. Consider questions such as:

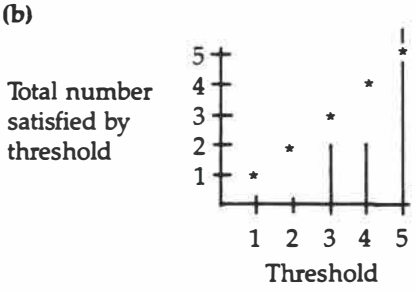
> What have you learned about the group as a result of this analysis? Are the methods of threshold analysis well-suited to analyzing this situation? If you were to start over, what would you do differently to obtain better results?

Solutions to "You Try It"

1. (a) Here is the completed table of threshold information, followed by the graph.

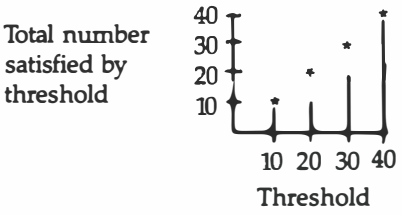
Threshold	Number holding threshold	Total number satisfied by threshold
1	0	0
2	0	0
3	2	2
4	0	2
5	3	5
None	0	

(b)

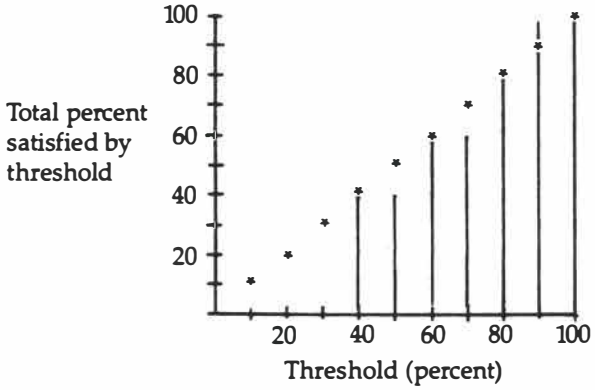


(c) All of the girls will be satisfied to go out for track if they do it all together. One can imagine a variety of different scenarios after that. Perhaps one or two will not like it and will want to drop out; perhaps several others will really like it and will stick with it no matter what the others do.

2. The values that belong in the last column of the given table of haircut threshold information for the Conestoga football team are 10, 10, 30, and 30. A graph of the threshold information follows. From the graph we see that groups of size 10 or 30 could end up with short cuts. As before, however, the achievement of these results depends on publicity. If Lee and Dan, and others who like the new hairstyle, talk it up and let their teammates know that their thresholds will be met, then either size group is possible. Probably the 10 young men whose thresholds are 10 would be first to get haircuts, with the others following them later as the style gets more publicity.



3. The values (percents) that belong in the last column of the table of conservation threshold information for Millville residents are 0, 0, 0, 40, 40, 60, 60, 80, 98, and 98. A graph of the threshold information is given below. From the graph we see that groups of size 40, 60, 80, or 98% of the Millville residents could end up conserving. If town leaders publicize the need for conservation and if residents believe that others will join them in supporting restrictions on water use, one would expect that 98% of the community will cooperate. On the other hand, if the 2% who fail to support conservation restrictions are blatant in their violations, others may join them. Perhaps the community will need to impose fines on those who fail to conserve.



4. The values (percents) that belong in the last column of the table of homework-copying threshold information for Merton seniors are 5, 10, 20, 30, 50, 90, 90, 90, 90, and 90. A graph of the threshold information is given below. From the graph we see that groups of size 50 or 90 Merton seniors might end up copying homework. If, however, student council takes a leadership role in discouraging dishonesty and if other students support this policy, then it is reasonable to expect that such large groups as 50 or 90 students copying homework would be discouraged from forming and, instead, honest behavior would result.