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Life in the Fast Line

A Modeling Problem from the Grocery Store

M. Jayne Fleener

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The objective of this HiMAP Pull-Out Section is to introduce students to mathematical modeling as a form of problem solving. Several important reports have recommended the incorporation of mathematical modeling in the curriculum. A model may be a replication of an object—like a model boat or airplane. A theoretical model is a set of rules that represent an object or process. When the rules are expressed mathematically, a mathematical model has been developed.

Here, we will develop a mathematical model that helps decide how to assign employees to the check-out counters in a grocery store. A mathematical simulation of two methods of using checkers and baggers will be conducted. Random numbers will be used to simulate the rate of arrival of customers in the check-out area. The teacher may appoint committees to do certain You Try Its.

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How much time have you spent waiting in lines at the grocery store? Retail business owners don't want you to wait in line, since you may be offended and shop elsewhere.

Suppose you are the manager of a small grocery store with two employees. You must decide which of the following would result in the least wait-time for your customers: have each employee operate a separate cash register and thus have two check-out lines, or have one employee work as cashier while the other bags the groceries, thus having one line. Let's assume that earlier analysis has indicated that with two cashiers and no baggers, checking averages two minutes per customer. With one cashier and one bagger, checking averages one minute per customer. Which arrangement will result in the least average wait-time for your customers?

Experimentation with the two methods could be used to solve this problem. However, it is not always possible to solve real-life problems with experimentation. Experiments that require humans are not only impossible in many instances, but are inefficient in terms of expense and time, and may offend customers.

Using Random Numbers to Model the Problem

We can model the problem by using random numbers to simulate how many customers arrive at the check-out area each minute, then gather data for the two methods and calculate the average wait-time for each. By simulating arrivals, we can observe which method causes the least wait-time for our customers.

Suppose earlier analysis has provided the information on customer arrival rates shown in **Table 1**. For any given minute, the probability of no customers arriving at the check-out area is 0.40. The probability of one customer arriving is 0.30, and the probability of two arriving is 0.30. We will run a simulation for 30 minutes and then calculate the average wait-time. Random numbers from 1 to 100 will be used.

Since the probability of no customers arriving at the check-out area is 0.40, the numbers 1 to 40 will represent this case. The numbers from 41 to 70 will represent the 0.30 probability that there will be one arrival in a minute, and the numbers from 71 to 100 will represent the 0.30 probability that there will be two arrivals in a minute.

The assignment of random numbers to simulate arrivals is displayed in **Table 2**. Random numbers can be obtained by running a BASIC program as in **Figure 1**, or by using a random number table, as in **Figure 2**. When using a random number table, one may pick any starting place in the table and proceed up or down columns, or across rows.

Table 1.

Arrival Rates of Customers

Percentage of Minutes	Number of arrivals at the check-out area each minute
40%	0
30%	1
30%	2

Table 2.

Random Number Assignments

Random Number Range	Number of arrivals at the check-out area each minute
1–40	0
41–70	1
71–100	2

Figure 1. Program to generate random numbers in BASIC, in both PC (top) and Apple versions.

PC Version

```
10 REM PROGRAM TO GENERATE NUMBERS FROM 1 TO 100
20 RANDOMIZE
30 MAX = 100
40 PRINT "How many random numbers do you wish to generate?"
50 INPUT NUM
60 FOR I = 1 TO NUM
70 REM GENERATE A NUMBER FROM 1 TO 100
80 RN = INT(RND*100+1)
90 PRINT RN,
100 NEXT I
110 END
```

Apple Version

```
10 REM PROGRAM TO GENERATE NUMBERS FROM 1 TO 100
20 MAX = 100
30 PRINT "How many random numbers do you wish to generate?"
40 INPUT NUM
50 FOR I = 1 TO NUM
60 REM GENERATE A NUMBER FROM 1 TO 100
70 RN = INT(RND*100+1)
80 PRINT RN,
90 NEXT I
100 END
```

91 76 21 64 64	44 91 13 32 97	75 31 62 66 54	84 80 32 75 77
00 97 79 08 06	37 30 28 59 85	53 56 68 53 40	01 74 39 59 73
36 46 18 34 94	75 20 80 27 77	78 91 69 16 00	08 43 18 73 68
88 98 99 60 50	65 95 79 42 94	93 62 40 89 96	43 56 47 71 66
04 37 59 87 21	05 02 03 24 17	47 97 81 56 51	92 34 86 01 82
63 61 06 34 41	94 21 78 55 09	72 76 45 16 94	29 95 81 83 83
78 47 23 53 90	34 41 92 45 71	09 23 70 70 07	12 38 92 79 43
87 68 62 15 43	53 14 36 59 25	54 47 33 70 15	59 24 48 40 35
47 60 92 10 77	88 59 53 11 52	66 25 69 07 04	48 68 64 71 06
56 88 87 59 41	65 28 04 67 53	95 79 88 37 31	50 41 06 94 76
02 57 45 86 67	73 43 07 34 48	44 26 87 93 29	77 09 61 67 84
31 54 14 13 17	48 62 11 90 60	68 12 93 64 28	46 24 79 16 76
28 50 16 43 36	28 97 85 58 99	67 22 52 76 23	24 70 36 54 54
63 29 62 66 50	02 63 45 52 38	67 63 47 54 75	83 24 78 43 20
45 65 58 26 51	76 96 59 38 72	86 57 45 71 46	44 67 76 14 55
39 65 36 63 70	77 45 85 50 51	74 13 39 35 22	30 53 36 02 95
73 71 98 16 04	29 18 94 51 23	76 51 94 84 86	79 93 96 38 63
72 20 56 20 11	72 65 71 08 86	79 57 95 13 91	97 48 72 66 48
75 17 26 99 76	89 37 20 70 01	77 31 61 95 46	26 97 05 73 51
37 48 60 82 29	81 30 15 39 14	48 38 75 93 29	06 87 37 78 48

Collecting and Organizing Data

Figure 3 is used to organize data for the case where there is only one check-out line with one employee as cashier and the other bagging groceries. Thirty random numbers were generated and are listed in the second column of Figure 3. The first column lists the minutes 1 to 30. As indicated, during the first minute, two people arrived to check out, since the random number 84 was given. Customers were numbered in the order of their arrival. Since Customer 1 arrived first, s/he was served while Customer 2 had to wait. Observe the data for the thirty minutes and the thirty random numbers assigned.

Figure 4 is used to simulate the case where there are two check-out lines, two cashiers, and no baggers. This table is similar to Figure 3, except that since there are two checkers, customers are assigned to the first one who is unoccupied. In the first minute, two customers arrived at the check-out area. Customer 1 was served in line 1, and Customer 2 was served in line 2. The same random numbers as in Figure 3 were used. Note that each customer now takes two minutes to be served, so in minute 4, Customer 3 would still be at line 1, while Customer 4 could then be served at line 2.

Calculating Average Wait-time

Average wait-time can be defined as the average number of minutes spent waiting in line for all customers arriving at the check-out area. (There are other ways to define average wait-time.) For our method, divide the total number of minutes waiting by the number of customers who arrived in line during the simulation.

In Figure 3, for the case where there was only one check-out line, the average wait-time would be $41 \div 30 = 1.37$ minutes or 1 minute and 22 seconds. For the two check-out line case in Figure 4, the average wait-time would be $40 \div 30 = 1.33$ minutes or 1 minute and 20 seconds.

In order to investigate the grocery store problem, do the following You Try Its. Number the problems and label your answers. ❑

Figure 3. Simulation of One Check-out Line.

Minute Number	Random Number	Number of Arrivals	Customer Arriving	Customer Being Served	Customer Waiting	Total Minutes Waiting
1	84	2	*1, 2	*1	*2	1
2	13	0	—	2	—	0
3	52	1	3	3	—	0
4	91	2	4, 5	4	5	1
5	67	1	6	5	6	1
6	42	1	7	6	7	1
7	12	0	—	7	—	0
8	24	0	—	—	—	0
9	67	1	8	8	—	0
10	73	2	9, 10	9	10	1
11	19	0	—	10	—	0
12	71	2	11, 12	11	12	1
13	48	1	13	12	13	1
14	81	2	14, 15	13	14, 15	2
15	91	2	16, 17	14	15, 16, 17	3
16	15	0	—	15	16, 17	2
17	44	1	18	16	17, 18	2
18	36	0	—	17	18	1
19	100	2	19, 20	18	19, 20	2
20	52	1	21	19	20, 21	2
21	8	0	—	20	21	1
22	88	2	22, 23	21	22, 23	2
23	62	1	24	22	23, 24	2
24	90	2	25, 26	23	24, 25, 26	3
25	21	0	—	24	25, 26	2
26	79	2	27, 28	25	26, 27, 28	3
27	47	1	29	26	27, 28, 29	3
28	39	0	—	27	28, 29	2
29	18	0	—	28	29	1
30	45	1	30	29	30	1

Figure 4. Simulation of Two Check-out Lines.

Minute Number	Random Number	Number of Arrivals	Customer Arriving	Line 1	Line 2	Customer Waiting	Total Minutes Waiting
1	84	2	*1, 2	1	2	—	0
2	13	0	—	1	2	—	0
3	52	1	3	3	—	—	0
4	91	2	4, 5	3	4	5	1
5	67	1	6	5	4	6	1
6	42	1	7	5	6	7	1
7	12	0	—	7	6	—	0
8	24	0	—	7	—	—	0
9	67	1	8	8	—	—	0
10	73	2	9, 10	8	9	10	1
11	19	0	—	10	9	—	0
12	71	2	11, 12	10	11	12	1
13	48	1	13	12	11	13	1
14	81	2	14, 15	12	13	14, 15	2
15	91	2	16, 17	14	13	15, 16, 17	3
16	15	0	—	14	15	16, 17	2
17	44	1	18	16	15	17, 18	2
18	36	0	—	16	17	18	1
19	100	2	19, 20	18	17	19, 20	2
20	52	1	21	18	19	20, 21	2
21	8	0	—	20	19	21	1
22	88	2	22, 23	20	21	22, 23	2
23	62	1	24	22	21	23, 24	2
24	90	2	25, 26	22	23	24, 25, 26	3
25	21	0	—	24	23	25, 26	2
26	79	2	27, 28	24	25	26, 27, 28	3
27	47	1	29	26	25	27, 28, 29	3
28	39	0	—	26	27	28, 29	2
29	18	0	—	28	27	29	1
30	45	1	30	28	29	30	1

You Try It #1

Run a simulation of the one check-out line case. To do this, use the blank Figure 5. Use a table of random numbers or a program to generate 30 random numbers in the range 1 to 100 and enter them in Figure 5 as described above. Then, as illustrated in Figure 3, complete Figure 5 and calculate the average wait-time to the nearest hundredth of a minute. Check your work because it will be used by the other students.

You Try It #2

Run a simulation similar to the two check-out lines case using the blank **Figure 6** and the same random numbers you used in You Try It #1. Calculate the average wait-time to the nearest hundredth of a minute. Again, check your work.

You Try It #3

Compare your results for the two cases. Which check-out method seems best? Build a table that labels and displays both average wait-times for the two methods and that displays the difference. Write a discussion.

You Try It #4

Combine your results from You Try It #1 with those of the other students in your class. Your teacher may want to organize this activity. Complete the following steps: a) Make a list from the largest to the smallest of the average wait-times calculated by your class. Check students' work. b) Give the largest, smallest, range, and median of the average wait-times. c) Calculate the average wait-time for the combined simulations. To do this, calculate the total number of customers for all simulations. Calculate the total number of minutes spent waiting for all simulations. Calculate the grand average wait-time as (total number of minutes waiting) ÷ (total number of customers waiting). Write a discussion and report your results. d) Do you think the larger simulation will yield more reliable results? Explain.

You Try It #5

As in You Try It # 4, combine your results from You Try It #2 with those of the rest of your class. Follow the steps suggested. Write a discussion and report your results.

You Try It #6

a) Compare the total class results from You Try Its #4 and #5 for the two cases. Which check-out method seems best? Build a table displaying for both cases the descriptive statistics that you think are interesting or useful. Write a review of the results. b) Did the total class results and the individual student results always agree on the best method? Which results are the most reliable? Write a discussion.

You Try It #7

Discuss with your classmates or your teacher other simulations that might be run for the grocery store, or for some other business or activity. You may want to define a problem and conduct a simulation. Some possibilities for the grocery store problem are: a) Suppose that a different definition of average wait-time is used. b) Suppose that average line-length is used. c) Suppose that the times required for checking and bagging are different from those given above. d) Suppose one checker is faster than another. e) Suppose that customer arrival rates are different for different times of the day. Some other projects that have been suggested are average wait-time for traffic lights, meals at a restaurant, ticket sales for a concert, and teller lines at a bank.

Figure 5. Blank table for one check-out line.

Minute Number	Random Number	Number of Arrivals	Customer Arriving	Customer Being Served	Customer Waiting	Total Minutes Waiting

Figure 6. Blank table for two check-out lines.

Minute Number	Random Number	Number of Arrivals	Customer Arriving	Line 1	Line 2	Customer Waiting	Total Minutes Waiting